Health care and intra-family decisions*

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!!VERY PRELIMINARY!!

Abstract

How does a family adapt if one of the members needs health care? The impact of the intra-household decision process of such a situation is usually not investigated. In this paper, we modelize the family as a set of single decision makers. Theoretically, we adapt a collective model to have a look inside the household, and measure the behavioural changes of all the family members when the family equilibrium becomes perturbated due to one member's health situation.

We consider households composed of two spouses, one of them needing health care. We suppose that the health of the care receiver is a household public good. We introduce a household production function, which depends on the time devoted to caregiving by the healthy spouse (informal care), and the money spend for the health care (formal care). In this way, we measure the productivity of caregiving.

We illustrate and test the performance of the theoretical model in using a synthetic data. A second part shall give empirical evidence for Germany from the 2000-2004 waves of German socio economic panel. This is still work in progress. The final aim is to simulate the impact of the recent health care reform in Germany, which consists in liberazing the health arrangements.

Key Words: domestic production, time use, intra household allocations, health care giving
JEL Classification: D11, D13, I12, J14, J22

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1 Introduction

The aim of our study is to measure the impact of health care need of one family member on the household decisions. The illness of one of the family member may have indeed severe consequences on both household's income and time use. On the one hand, the one who is ill may reduce his activities on the labour market, as well as inside the household. On the other hand, the healthy members of the family have to increase their contribution to the housework and to assist and take care of the ill person. Beside the impact on time use, the household income may brake down due to a lower or a non-participation of the ill person on the labour market. Furthermore, expenses for health and health care may increase sharply. This may not be compensated by a revenue from health insurance.

We investigate this question in adapting a collective model with household production initially proposed by Bourguignon and Chiuri (2005). Household production was first introduced into an economic household model by Chiappori (1997) and Apps and Rees (1997). They show that in the collective framework the allocation of time between market work and household production matters, and therefore the true consumption of leisure cannot be derived from the sole observation of market time. Some new results have been obtained recently, in the case of marketable (Bourguignon and Chiuri 2005) as well as in the case of non marketable goods (for both cases Rapoport et al. 2003 and 2005, Rapoport and Sofer 2004). Donni (2004) shows that, even if domestic time inputs are not observed, valid welfare comparisons are still possible. However strong restrictions have to be imposed on the home production function (constant return of scale) as well as on the preferences structure (separability). Bourguignon and Chiuri (2005) estimate individual domestic productivity, starting from the informational content of a time use survey, at least whenever household members are observed working on the labour market. They find that female domestic productivity is a relevant variable for explaining the intra-household distribution of resources in a sample of French households.

In our study, the public good is the health of the ill spouse. The latter depends for part on the time devoted to healthcaring and the expenditures for health.

The novelty of our study is that we develop a collective model in which both spouses do not have a labour market activity. The identification of the model is based on the specification of the housework supply functions, instead of traditionnally the labour supply functions. In order to obtain variability of the income over the housework time, which is necessary to obtain the identification of the model, we impute wages using the information on retirement pension, or former labour market income. This wages measure the productivity of each family member regarding the household work.

We illustrate and test the performance of the theoretical model in using a synthetic data. We propose two estimation procedures: we estimate a model specifying continuous housework functions, adapting for our purpose the model proposed by Bourguignon and Chiuri (2005). The second estimation strategy is a discrete choice approach, adapted from Beninger (2006).

We intend to use the 2000-2004 waves from the German socio-economic panel (GSOEP) to give empirical evidence for Germany. This is still work in progress. Final aim is to simulate the impact of the recent health care reform in Germany, which consistes in liberazing the health arrangements.

The paper is organised as follows. In Section 1, we describe the model. First, we present the model with household production we use. Then, we show how we identify the household parameters. Section 2 presents the two estimation procedures we use. In Section 3, we describe the data and give some statistical evidence. Section 4 contains the estimation results.Finally, Section 5 concludes.

2 The model

In our study, we are interested in the intra-household aspects of a change of the health situation of one of the spouses. Therefore, we represent the family as a negotiating set of individual decision makers. We use a collective model.

Families in which one of the spouses needs health care are usually composed of elderly people. Thus, in our data, the spouses do not work . This is a problem for the identification of the model. In a first subsection, we present our model. In a second subsection, we specify the functional forms and the way we identify the household parameters.

2.1 A general representation

The decision within the family is supposed to be Pareto-optimal. The couple produces a public good d. Therefore the couple solves following maximization problem:

$$\max_{c_f,l_f,c_m,l_m,d,h_f,h_m,c_h} U^f\left(c_f,l_f,c_m,l_m,d\right) \tag{1}$$

$$s.t. \begin{cases} U^m\left(c_m,l_m,c_f,l_f,d\right) \geq \overline{U}_m \\ c_f+c_m+c_h \leq y \\ d=f\left(h_f,h_m,c_h,\pi_f,\pi_m,H_i\right) \\ h_f+l_f+l_{fd}+l_0\left(H_f\right)=T \\ h_m+l_m+l_{md}+l_0\left(H_m\right)=T, \end{cases}$$

where $U^i\left(c_f,l_f,c_m,l_m,d\right)$ represents the utility. c_i and l_i are the consumption and the leasure of spouse i. y is the couple unearned income. Note that y comprises rental and capital revenues (and other unearned incomes) as well as retirement pension and paiement from the healthcare insurance y_h. The latter depends on the degree of handicap H_i of the ill spouse. We pose that $H_{j,j\neq i}=0$, i.e. only one spouse needs health care. $\overline{U}_m$ is the negotiated utility level reached by the husband at optimum. f represents the production function for the public good. In our case, the public good is the health of the ill spouse. d depends on the time devoted by the one spouse giving care h_i, by the supplement money input for health care c_h and H_i. π_f and π_m

represent the female and the male productivity in giving health care. Note that only one spouse (the healthy one) has a positive time for ousehold production ($h_i \geq 0, h_{j,j \neq i} = 0$). However, we suppose that female and male have different productivities, and a different structure in time-use: the devotion to the spouse may differ whether one considers the wife or the husband. We therefore separate the household production function in a female and a male component (see also equation (3)). T is the total available time. l_{id} is the time spouse i spends for other housework (for example cleaning, gardening, etc.). l_0 is the regeneration time, which depends on the handicap degree of the person.

2.2 Identification and functional specification

Due to the subject of our study (health care), we consider elderly people, i.e. people who are not more active on the labour market. The problem is that a collective model is identified thanks to the observation of the spouses' labour supplies and the household consumption. Basically, the identification procedure of a collective model uses the variation, through the population, of the labour supplies regarding the wages and the income structure of the couple (income from labour vs. unearned income) to obtain the identification of the individual preferences and the intra-household decision process (income sharing rule) (see Chiappori 1988). This methodology can obviously not been applied here.

To overcome the identification problem, we use the housework supply function instead traditionnally the labour supply functions. Of course, time devoted to housework is not paid. However, to obtain any housework time depending variation in the budget constraint, we suppose that each spouse is virtually remunerated for his effort. This virtual salary $\tilde{w}_i$ measures his abality in doing housework. $\tilde{w}_i$ may be calculated using the information on the former salary or may be imputed from the estimation of a wage equation, using the information on the indivuals characteristics. Therefore, the budget constraint is now:

$$c_f + c_m + c_h \leq \tilde{w}_f \left(h_f + l_{fd} \right) + \tilde{w}_m \left(h_m + l_{md} \right) + y. \quad (2)$$

The domestic production function is:

$$\begin{aligned} d &= f \left(h_f, h_m, c_h, \pi_f, \pi_m, H \right) \quad (3) \\ &= \left(\theta_f h_f + \pi_f \right)^{\gamma_f} + \left(\theta_m h_m + \pi_m \right)^{\gamma_m} + \left(\theta_h c_h \right)^{\gamma_h} - \theta_H H, \end{aligned}$$

where $(\gamma_i)_{i=f,m} \in]0, 1[$. The function $f(.)$ has two separable components which measure men's and women's contributions to home production and both satisfying the property of decreasing return to scale. The additive separability hypothesis excludes cases of joint production. The first order condition leading to a positive time spent in domestic production is:

$$\gamma_i \theta_i \left(\theta_i h_i + \pi_i \right)^{\gamma_i - 1} = w_i, \quad (4)$$

(see Bourguignon and Chiuri 2005 for further details). Note that it may be difficult to consider the money input c_h: this information is usually not provided in the data set.

We can now identify the household parameters (paramaters of the individual preferences, parameters of the intra-household sharing rule, parameters of the household production function) once we have specified the functional form of the preferences and either of the housework supply functions or the income sharing rule compatible with the collective rationality.

Note that the difficulty of the identication is due to non-observable labour market time. Here the budget constraint does not depend on the tax-benefit system. Non-participation is not really a problem, since most of the people in a data set indicate a positive value for housework time, except for heavily ill person though.

3 Estimation procedure

3.1 Continuous specification

The functional form of the housework supply functions is given in equation (5):

$$U^i\left(c_f, l_f, c_m, l_m, d\right) = a_c^i \ln\left(c_i - c_0\right) + a_l^i \ln\left(l_i - l_0\right) + a_d d$$

We pose that the functional form for the housework supply functions ($L_i = h_i + l_{id}$) :

$$\begin{aligned} L_i &= \beta_o^i + \beta_1^i \ln w_f + \beta_2^i \ln w_m + \beta_3^i \ln w_f \ln w_m + \beta_4^i \left(\ln w_f - \ln w_m\right) \\ &\quad + \beta_5^i y + \beta_6^i y^2 + \beta_7^i \pi_f + \beta_8^i \pi_m + \gamma \Gamma, \end{aligned} \tag{5}$$

where Γ is a set of household characteristics. Γ may comprise the handicap degree H.

The income sharing rule Υ is obtained by second and third order partial derivatives from functions L_i:

$$\begin{aligned} \Upsilon &= k + \frac{\beta_3^f \beta_1^m}{D_b} \ln w_f + \frac{\beta_2^f \beta_3^m}{D_b} \ln w_m + \frac{\beta_3^f \beta_3^m}{D_b} \ln w_f \ln w_m + \frac{\beta_4^f \beta_3^m}{D_b} \left(\ln w_f - \ln w_m\right) \\ &\quad + \frac{\beta_5^f \beta_3^m}{D_b} y + \frac{\beta_6^f \beta_3^m}{D_b} y^2 + \frac{\beta_7^f \beta_1^m}{D_b} \pi_f + \frac{\beta_8^f \beta_1^m}{D_b} \pi_m + \frac{\gamma \beta_1^m}{D_b} \Gamma, \end{aligned} \tag{6}$$

where $D_b = \beta_6^f \beta_3^m - \beta_3^f \beta_6^m$. One can also derive testable restriction (see Appendix A).

Using continuous housework supply functions does not allow us to introduce any 'caring' aspect in the preferences beyond the health of the ill spouse.

We use a general method of moments to calculate the household parameters.

3.2 Discrete choice specification

Alternatively, we use a discrete choice approach, as proposed by Beninger (2006).

Suppose that the male utility function is invertible w.r.t. the male consumption. Therefore the male consumption c_m is the solution of following equation:

$$c_m = W^m\left(\overline{U}_m, l_m, c_f, l_f, d\right). \tag{7}$$

The maximization problem is:[1]

$$\max_{c_f,l_f,l_m,d,h_f,h_m,c_h} U^f\left(c_f, l_f, W^m\left(\overline{U}_m, l_m, c_f, l_f, d\right), l_m\right) \tag{8}$$

$$s.t. \begin{cases} c_f + c_m + c_h \leq \tilde{w}_f\left(h_f + l_{fd}\right) + \tilde{w}_m\left(h_m + l_{md}\right) + y \\ d = f\left(h_f, h_m, c_h, \pi_f, \pi_m, H_i\right) \\ h_f + l_f + l_{fd} + l_0\left(H_f\right) = T \\ h_m + l_m + l_{md} + l_0\left(H_m\right) = T. \end{cases} . \tag{9}$$

In order to avoid problems in the estimation with differences in the utility level between the individuals, we turn to introduce the "male's power index". For that we define ω_m, which is defined as the normalized utility the reaches at the optimum:

$$\omega_m = \frac{\overline{U}_m - U_m^{\min}}{U_m^{\max} - U_m^{\min}}, \tag{10}$$

where $\overline{U}_m$, $U_m^{\min}$ and $U_m^{\max}$ are respectively the effective, minimum and maximum utility levels the husband can reach. Define c as the household consumption: $c = c_f + c_m$. Note that the consumption share $\rho = \frac{c_f}{c}$ and ω_m are equivalent measures of the allocation rule of the resource within the household. This means that, in case of a parametric specification of the estimation equations, the expression of the one can be deduced from the specification of the other equation. However, in order to avoid complex computations, we impose the restriction that the individual preferences do not depend on the spouse's consumption. Equation (8) is then:

$$\begin{aligned} & \max_{c,l_f,l_m} U^f\left(c - W^m\left(\overline{U}_m, c, l_m, l_f, d\right), l_f, l_m\right) \\ \Longleftrightarrow \quad & \max_{c,l_f,l_m} \tilde{U}^f\left(c, \overline{U}_m, l_m, l_f, d\right) \\ \Longleftrightarrow \quad & \max_{c,l_f,l_m} \hat{U}^f\left(c, \omega_m, l_m, l_f, d\right). \end{aligned} \tag{11}$$

For the estimation, we assume that each individual has $n = 6$ possible choices for his or her weekly housework supply: $l_{id} = 0, 10, \ldots, 50$. This yields a set of $N = 36$ choices for the housework suplly pairs (l_{fd}, l_{md}) of the spouses. If $\hat{U}\left(c^j, \omega_m^j, l_f^j, l_m^j\right)$ denotes the utility generated by combination $\left(c^j, \omega_m^j, l_f^j, l_m^j\right)$, adding an error term ε_j, we define actual utility derived from combination j as:

$$\hat{U}_j^f = \hat{U}^f\left(c^j, \omega_m^j, l_f^j, l_m^j, d\right) + \varepsilon_j \qquad \forall\, j = 1, ..., N. \tag{12}$$

Note that the male power index ω_m^j depends on each spouse's contribution to the household wealth (see below). Specifying the extreme value distribution for ε_j, defined by:

$$\Pr\left[\varepsilon_j < \varepsilon\right] = \exp\left(-\exp\left(-\varepsilon\right)\right), \tag{13}$$

[1] $\overline{U}_m$ is not a choice variable. The allocation rule of the ressources is defined a priori by the couple.

leads to the multinomial logit model:

$$\Pr\left[\hat{U}_j^f > \hat{U}_k^f, \forall\, k \neq j\right] = \frac{\exp\left(\hat{U}^f\left(c^j, \omega_m^j, l_f^j, l_m^j, d\right)\right)}{\sum_{k=1}^{N} \exp\left(\hat{U}^f\left(c^k, \omega_m^k, l_f^k, l_m^k, d\right)\right)}. \tag{14}$$

Expression (14) represents the household's contribution to the likelihood.[2] Note that $U_m^{\min}$ and $U_m^{\max}$ are the minimum and maximum utility the man can reach over all alternatives.

Specifically, we suppose that the preferences are:

$$U^i = a_c^i \ln(c_i - c_0) + a_l^i \ln(l_i - l_0) + a_{ll}^i \ln(l_f - l_0)\ln(l_m - l_0) + \delta f(.), \tag{15}$$

where c_0 is the minimum consumption. l_0 corresponds to the time devoted to regeneration. Both c_0 and l_0 depend on the number of children in the household. We have to impose the linearity in the consumption term in the preference equation. A quadratic specification (or a specification of higher power) would lead to multiple solutions for equation (7). A cross consumption-leisure term $\ln(c_i - c_0)\ln(l_i - l_0)$ unnecessarily complicates the estimation procedure.

The econometric specification of the male power index ω_m, i.e. the intra-household allocation rule of the resources, is:

$$\omega_m = \frac{1}{1 + \exp \mathbf{B}}, \tag{16}$$

where $\mathbf{B} = b_0 + b_{w_f} \ln w_f + b_{w_m} \ln w_m + b_{ww} \ln w_f \ln w_m + b_y y + b_{w_f y} \ln w_f y + b_{w_m y} \ln w_m y + b_{y^2} y^2 + \mathbf{b_d \Gamma}$. $\mathbf{\Gamma}$ is a $m-$vector of socio-demographic variables and distribution factors. Using equation (10) and equation (15), we obtain:

$$\begin{aligned} c_m &= \exp\left(\frac{\omega_m\left(U_m^{\max} - U_m^{\min}\right) + U_m^{\min} - a_l^m \ln(l_m - l_0) - a_{ll}^m \ln(l_f - l_0)\ln(l_m - l_0) - \delta f(.)}{a_c^m}\right) \\ &= V_m + c_0. \end{aligned} \tag{17}$$

Finally, we estimate the following equation:

$$U^f = a_c^f \ln(c - V_m) + a_l^f \ln(l_i - l_0) + a_{ll}^f \ln(l_f - l_0)\ln(l_m - l_0) + \delta f(.). \tag{18}$$

All the parameters $a_{j=c,l,ll}^{i=f,m}$ and b are identified separately if we assume that the utility stemming from common leisure is the same for both spouses, i.e.: $a_{ll}^f = a_{ll}^m$.

We estimate this model using conditional logit.

4 Data and estimation results: synthetic data

4.1 Data generating process

We generate data for 2,000 couples (see Appendix B).

[2] We have tried a mixed multinomial estimation. We introduced a mass point coefficient on the consumption variable, allowing for heterogeneity on consumption among the families (see Hoynes 1996). However the maximum likelihood procedure did not more converge. We did not try to introduce random coefficients yet (Train 2000).

Table 1: Summary statistics: data with home production

		min	10%	50%	90%	max	st. dev.
c_f	female consumption	298	1700	4720	10230	25223	3652
c_m	male consumption	306	2080	5308	11870	54273	3996
c_d	input for the household good	0	238	1240	2410	5741	302
l_f	female leisure per week	88.0	91.2	125.1	157.6	168.0	
l_m	male leisure per week	88.0	92.7	129.4	155.2	168.0	
h_f	female time devoted to health care	0.0	0.0	16.8	54.2	80.0	
h_m	male time devoted to health care	0.0	0.0	0.0	49.7	75.7	
l_{fd}	female time devoted to housework	1.0	15.9	50.4	68.3	78.1	
l_{md}	male time devoted to housework	1.0	12.7	46.0	66.7	78.9	
k_f	female power index	.203	.339	.556	.788	.961	.147
k_m	male power index	.277	.498	.603	.703	.977	.104
α	convexity coeff. of Pareto-frontier	1.00	1.025	1.184	1.813	4.110	.386

4.2 Data: statistics

Table 1 summarizes the family decisions in the egoistic, 'caring' and home production cases respectively. The female consumption and leisure demand is lower than male's due to a lower wage rate. When considering home production, the wives devote more time to the production of the household good: the relative cost is lower. Remarkable is also that the difference between female and male behaviour is lower when allowing for altruism or in the case of home production.

Figures 1 and 2 show the distribution of the convexity coefficient of the Pareto-frontier α and the male normalized power index in the egoistic case.[3] The Pareto-frontier is almost linear for a majority of households. The distribution of κ_m is slightly squewed to the left, even if the average of κ_m is greater than .5, denoting that males are in average favoured.

4.3 Estimation

The estimation is in two steps. First, we estimate the household productivity (Equation (3)). We compute the fitted productivies $\hat{\pi}_i$. We then able to estimate housework supply functions (continuous specification) or the parameters of the discrete choice model. Tables 2, 3 and 4 show the parameter estimates for the female and male housework supply functions (continuous model) and the estimates for the income sharing rule as well as the preference estimates (discrete choice model).

Table 5 gives the mean-absolute errors for the prediction of the time devoted to health care and housework, for female and male. The discrete choice model seems to perform better than the model with continuous housework functions. The lower prediction errors for males may be

[3] The distributions are not very different in the other cases. They are not shown in order to save place.

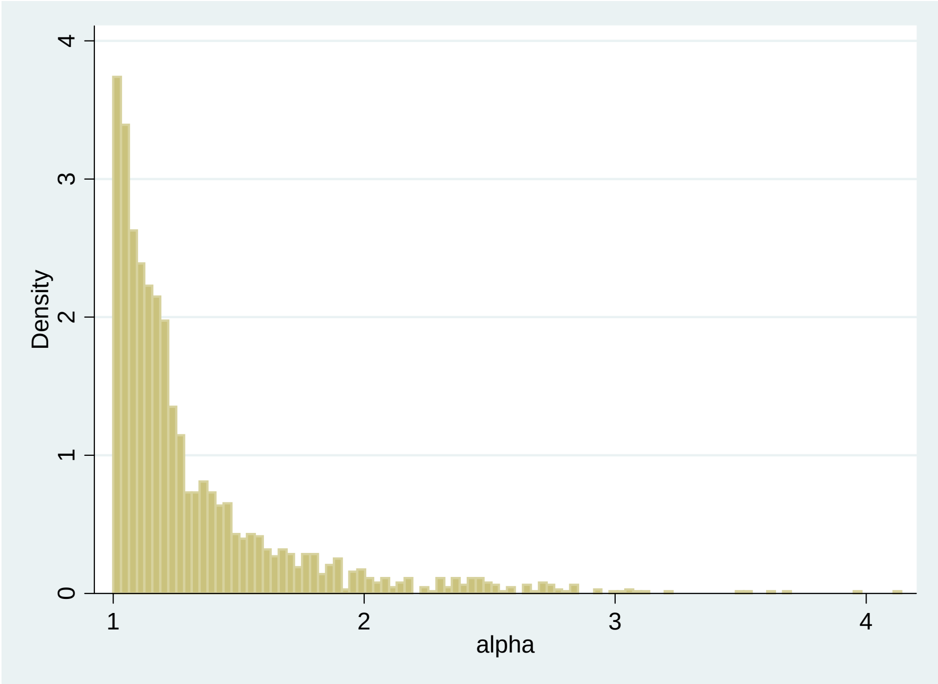

Figure 1: Distribution of the convexity coefficient of the Pareto-frontier

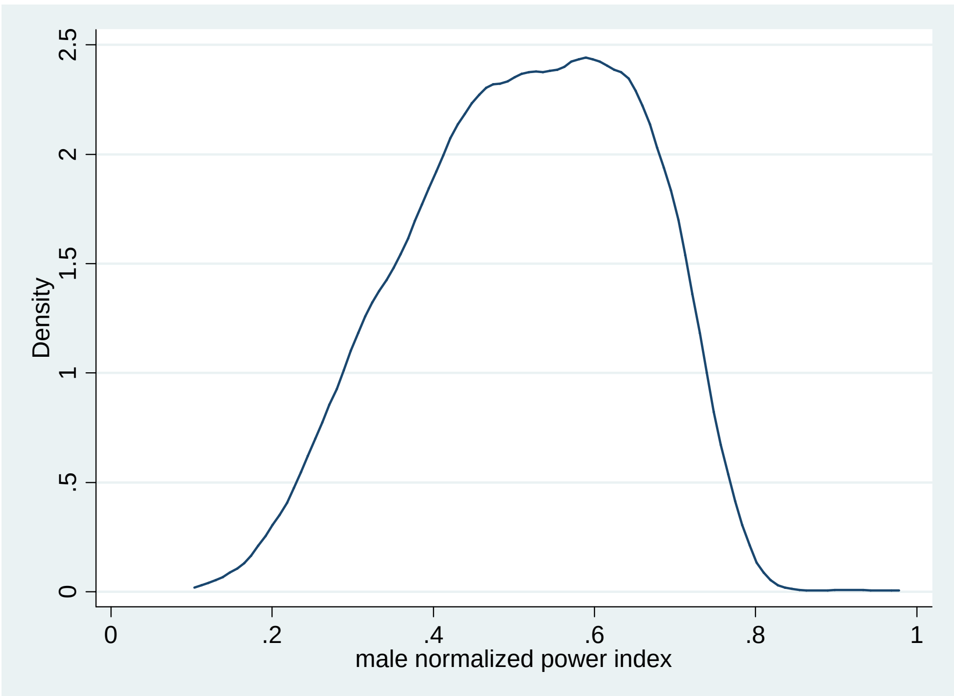

Figure 2: Epanechnikov kernel estimation of the distribution of the male normalized power index

Table 2: Housework supply estimation - female - continuous specification

parameter	variable	variable name	coef.	s.d.
β_o^f	$-$	constant	7.41	
β_1^f	$\ln w_f$	log female wage	1.23	
β_2^f	$\ln w_m$	log male wage	-.452	
β_3^f	$\ln w_f \ln w_m/10$	cross log wages	.015	
β_4^f	$\ln w_f - \ln w_m$	log wages difference	.087	
β_5^f	$y/100$	unearned income	.015	
β_6^f	$y^2/10000$	unearned income squarred	.001	
β_7^f	π_f	female productivity	.125	
β_8^f	π_m	male productivity	-.018	
γ^f	Γ	distribution variable	.103	

Table 3: Housework supply estimation - male - continuous specification

parameter	variable	variable name	coef.	s.d.
β_o^m	$-$	constant	6.25	
β_1^m	$\ln w_f$	log female wage	-.287	
β_2^m	$\ln w_m$	log male wage	.601	
β_3^m	$\ln w_f \ln w_m/10$	cross log wages	.010	
β_4^m	$\ln w_f - \ln w_m$	log wages difference	-.008	
β_5^m	$y/100$	unearned income	-.004	
β_6^m	$y^2/10000$	unearned income squarred	.001	
β_7^m	π_f	female productivity	-.028	
β_8^m	π_m	male productivity	.158	
γ^m	Γ	distribution variable	-.057	

Table 4: Parameter estimates - discrete choice model

parameter	variable	variable name	coef.	s.d.
b_0	$-$	constant	.912	
b_{w_f}	$\ln w_f$	log female wage	.521	
b_{w_m}	$\ln w_m$	log male wage	-.452	
b_{ww}	$\ln w_f \ln w_m/10$	cross log wages	.010	
b_y	$y/100$	unearned income	.024	
b_{y^2}	$y^2/10000$	unearned income squarred	.013	
γ	Γ	distribution variable	.102	
a_c^f	$\ln(c_f - c_0)$	female consumption	.476	
a_l^f	$\ln(l_f - l_0)$	male consumption	.498	
a_c^m	$\ln(c_m - l_0)$	female leisure	.512	
a_l^m	$\ln(l_m - l_0)$	male leisure	.486	
a_{ll}	$\ln(l_f - l_0)\ln(l_m - l_0)$	cross leisure	.094	
δ	$f(.)$	household production	.116	
θ_f	h_f	female health care	1.13	
θ_m	h_m	male health care	1.08	
θ_h	c_h	expenditure for health	.110	
θ_H	H	handicap degree	2.57	
γ_f		female propensity for health care	.457	
γ_m		male propensity for health care	.213	
γ_h		propensity for health expenditure	.059	

Table 5: Mean-absolute errors in time-use predictions for female and male

	health care		other housework	
	female	male	female	male
Continuous model	2.23	2.08	3.20	2.09
Discrete choice model	1.84	1.27	1.96	.75

due to a lower variability in male time use.

5 Conclusion

In this paper, we have extended the collective model to the case in which neither of the spouses works. We have also introduced health care. We have used two estimation procedures: the one using continuous housework supply functions. The second is a discrete choice model. We have shown that the latter performs better in terms of goodness of fit.

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6 Appendix B

6.1 Model specification

The couple solves the problem:

$$\max_{c_f,l_f,c_m,l_m,d,l_{fd},l_{md},c_d} U^f\left(c_f,l_f,c_m,l_m,d\right) \tag{19}$$

$$s.t. \begin{cases} U^m\left(c_m,l_m,c_f,l_f,d\right) \geq \overline{U}_m \\ c_f+c_m+c_h \leq \tilde{w}_f\left(h_f+l_{fd}\right)+\tilde{w}_m\left(h_m+l_{md}\right)+y \\ d=f\left(h_f,h_m,c_h,\pi_f,\pi_m,H_i\right) \\ h_f+l_f+l_{fd}+l_0\left(H_f\right)=T \\ h_m+l_m+l_{md}+l_0\left(H_m\right)=T, \end{cases}$$

where:

$$\begin{aligned} U^i &= a^i \ln(c_i - c_0) + (1 - a^i) \ln(l_i - l_0(H_i)) + \mu U_j + \delta d, \\ f(h_f, h_m, c_h, \pi_f, \pi_m, H) &= (\theta_f h_f + \pi_f)^{\gamma_f} + (\theta_m h_m + \pi_m)^{\gamma_m} + (\theta_h c_h)^{\gamma_h} - \theta_H H. \end{aligned} \tag{20}$$

Note that the preferences are 'caring'.

6.2 Definition of the parameters of the model

There are fixed parameter values which are common to all individuals (like total available time, for instance). The individual parameters are particular to each individual (like gross wage rates). In the latter case, the individual values result from i.i.d. sampling for each individual (for wage rates and for the marginal propensities to consume) and for each couple (unearned income). The fixed parameters, i.e. minimum consumption, $\bar{c}$, total time available, T, and regeneration time l_0, are common to the whole population. All pertain to a week. Minimum consumption per week is set to $\bar{c} = 200$ monetary units. Total time available, in hours, is the duration of a week: $T = 168$ hours. $l_0 = 68 + 25H$ corresponds to the regeneration time (sleeping, eating, e.g.), where $H = 0$ for the healthy spouse and $H = H_i$ for the ill spouse. We suppose there are three different levels for H_i: $H_i = 1, 2, 3$ and $p(H_i = 1) = .6$, $p(H_i = 2) = .3$ and $p(H_i = 3) = .1$. Maximum domestic production time is 60 hours a week.

Wage distributions are highly skewed to the right, and extremely low wages are rare in the rich countries (see e.g. Hildenbrand, 1994 and 1998). With these characteristics in mind, we specify the following density for the wages – this is a special case of the Fisk distribution (see e.g. McDonald, 1984):

$$f_{w_i}(w_i) = \frac{w_i}{d_i^2} \exp\left(-\frac{w_i}{d_i}\right) \mathbf{1}(w_i \geq 0) \qquad \forall\, i = f, m. \tag{21}$$

The corresponding cumulative probability function is:

$$\begin{aligned} F_{w_i}(w_i) &= \mathbf{1}(w_i \geq 0) \int_0^{w_i} \frac{t}{d_i^2} \exp\left(-\frac{t}{d_i}\right) dt \\ &= \mathbf{1}(w_i \geq 0) \left[1 - \left(\frac{w_i}{d_i} + 1\right) \exp\left(-\frac{w_i}{d_i}\right)\right] \end{aligned} \tag{22}$$

where $d_f = 40$ and $d_m = 50$. Expectation, standard error and mode are respectively $E(w_i) = 2d_i$, $\sigma(w_i) = d_i\sqrt{2}$, and mode$(w_i) = d_i$. Figure A1 depicts the resulting wage distributions.

For *unearned income* we assume:

$$y = y_f + y_m + .8 \times U40w_f + .8 \times 40w_m + y_u + y_h, \tag{23}$$

where y_f and y_m are the female and male retirement pension. We suppose that the retirement pension corresponds to 80% of the former work income. Males are supposed to have worked full

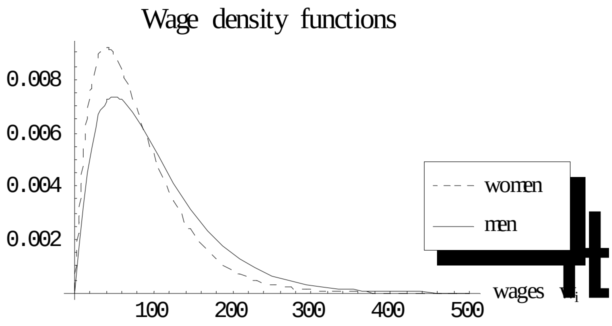

Figure 3: Wage densities

time ($y_m = .8 \times 40 w_m$). Female worked part-time ($y_f = .8 \times U 40 w_f$), where U is the uniform distribution. y_u is the household revenue from rent, capital, etc. We assume the exponential distribution:

$$f_y(y) = \frac{1}{\theta} \exp\left(-\frac{y}{\theta}\right) \mathbf{1}(y \geq 0), \tag{24}$$

with $\theta = 5000$. Expectation, standard error and mode are $E(y) = \sigma(y) = \theta$, and mode$(y) = 0$. y_h is the income from the health insurance. This depends on the level of handicap of the ill spouse H_i:

$$y_h = 1000 H_i. \tag{25}$$

We further suppose that the probability that the female take care of her husband is 70%.
The marginal propensity to consume (a_i) is from a normal distribution:

$$a_i \propto N(.5, .09). \tag{26}$$

The *altruism parameter* μ and the *preference parameter* δ for the domestic production follow a normal distribution with mean .2 and standard deviation .04:

$$\begin{aligned} \mu &\propto N(.2, .04), \\ \delta &\propto N(.2, .04). \end{aligned} \tag{27}$$

Parameters $\theta_{f,m,h,H}$ are extracted from normal distributions:

$$\begin{aligned} \theta_f &\propto N(1.2, .09) \\ \theta_m &\propto N(1.0, .09) \\ \theta_h &\propto N(.1, .004) \\ \theta_H &\propto N(3, .01) \end{aligned} \tag{28}$$

Note that the spouses are randomly married.

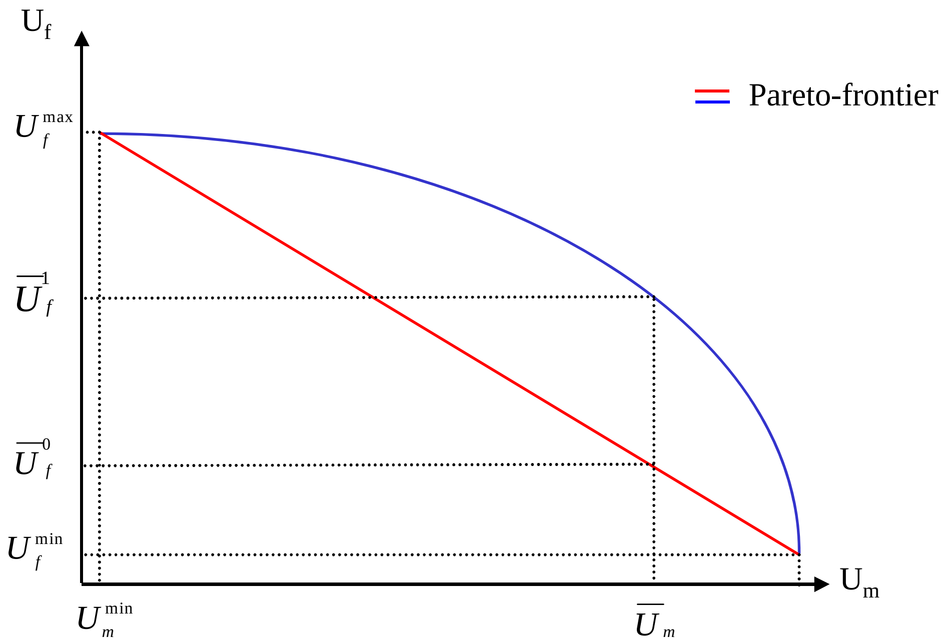

Figure 4: Concavity of the Pareto-frontier

6.3 Specification of the income sharing rule

Of course, the negotiated utility level reached by the husband at optimum $\overline{U}_m$ has also to be defined. However, in order to have a measure which is comparable for all couples, the relative reached utility k_m is used

$$k_m = \frac{\overline{U}_m - U_m^{\min}}{U_m^{\max} - U_m^{\min}}. \tag{29}$$

But as illustrated by Figure 4, k_m does not tell us a lot on the intra-family distribution of 'power'. Indeed suppose that $k_m = .8$. If the Pareto-fontier is linear (red line), the husband beneficiates relatively to his wife of a higher 'power' than if the Pareto-fontier is more concave (blue line): $\overline{U}_f^0 < \overline{U}_f^1$. Therefore a more appropriate measure is the *male power index* κ_m

$$\kappa_m = k_m^\alpha, \tag{30}$$

where α is such that $k_m^\alpha + k_f^\alpha = 1$. k_f is the relative female utility at the optimum: $k_f = \frac{\overline{U}_f - U_f^{\min}}{U_f^{\max} - U_f^{\min}}$. The power α can be considered as the concavity coefficient of the Pareto-frontier at the optimum. If $\alpha = 1$, the Pareto-frontier is linear. If $\alpha = 2$, the Pareto-frontier is a circle.

The functional form of the κ_m is:

$$\kappa_m = 1/(1 + exp(-aaa/.8)) \ , \tag{31}$$

$$\text{where} \ \ aaa = 3x - x^2 - y/10000 + .25(y/10000)^2 - .2xy/10000 - .8 + .1\Gamma \ ,$$

$$\text{where} \ \ x = 1/2(w_m/(w_f + w_m) + y_m/y) \ .$$

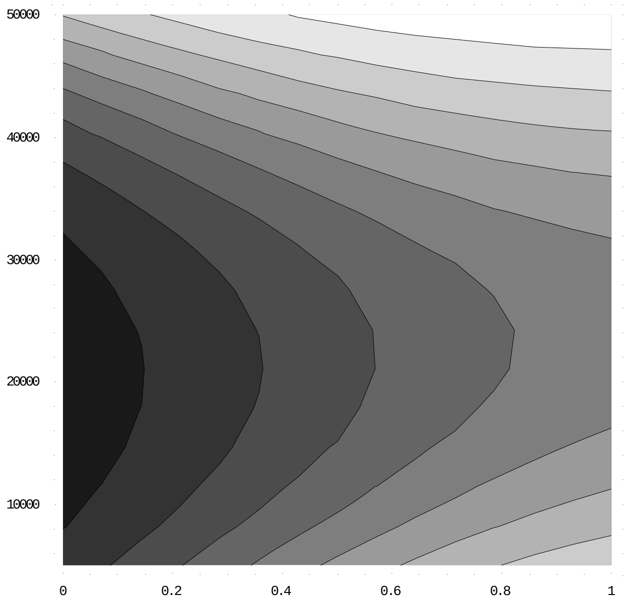

Figure 5: The male power index

The male power index κ_m depends on the household unearned income y and the relative contribution of the husband to the household wealth. Γ is a distribution variable, which is here randomly extracted from a uniform distribution. From Figure 5, we can see that the male power index increases with the husband's contribution to the household wealth x. If the household has relative low unearned income, κ_m increases with y. On contrary, κ_m decreases with y for families high non-earned income. In fact, only a very few families have an unearned income greater than 20,000 monetary unit.

In order to solve the maximization problem 19, the negotiated male utility level $\overline{U}_m$ has to be computed from κ_m. In the egoistic case, this is done in solving the following system of equation:

$$
\begin{aligned}
k_m &= (\kappa_m)^{\frac{1}{\alpha}} \\
k_f &= (1-\kappa_m)^{\frac{1}{\alpha}} \\
k_f &= \frac{\overline{U}_f\left|_{\overline{U}_m} - U_f^{\min}\right.}{U_f^{\max} - U_f^{\min}}.
\end{aligned} \tag{32}
$$

The third equation denotes that the intra-family decision is Pareto-optimal. System (32) has no analytical solution due to the complexity of latter. System (32) is solved using a Gauss-Newton iterative algorithm. The individual preferences are not separable. Therefore it is not possible to find any analytical solution to maximization problems 1. They are solved using a Gauss-Newton algorithm integrating the computation of $\overline{U}_m$.